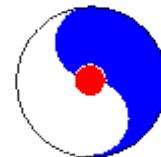


Sample of Lattice Activities

Taku Izubuchi



RIKEN BNL Research Center

Lattice Gauge theories

- Non-perturbative computations from the truly first principle
- Indispensable bridge between experiments and theories to find new laws in elementary particle physics. Precision frontier.
- Also explores new theories
- Symmetries are key ingredients (gauge symmetry, chiral symmetry of quarks)

~1973 QCD, '74 Lattice formulation

'79 Lattice Gauge simulation (M. Creutz)

....Spectrum, [Wilson Fermion](#), Kogut-Susskind fermion...

'97~ [Domain-Wall Fermions](#) (DWF) Nf=0 quenched

(Blum & Soni, CP-PACS,

'99~ Riken-BNL-Columbia (RBC))

'02~ DWF Nf=2 up, down quarks RBC

'05~ DWF Nf=2+1 up, down, strange quarks (all light quarks) RBC/UKQCD

'08~ Graphene fermion (M. Creutz)

QCDSP



CP-PACS ~1Tflops



QCDOC ~10Tflop BlueGene/L,P ~ 10-100 Tflops



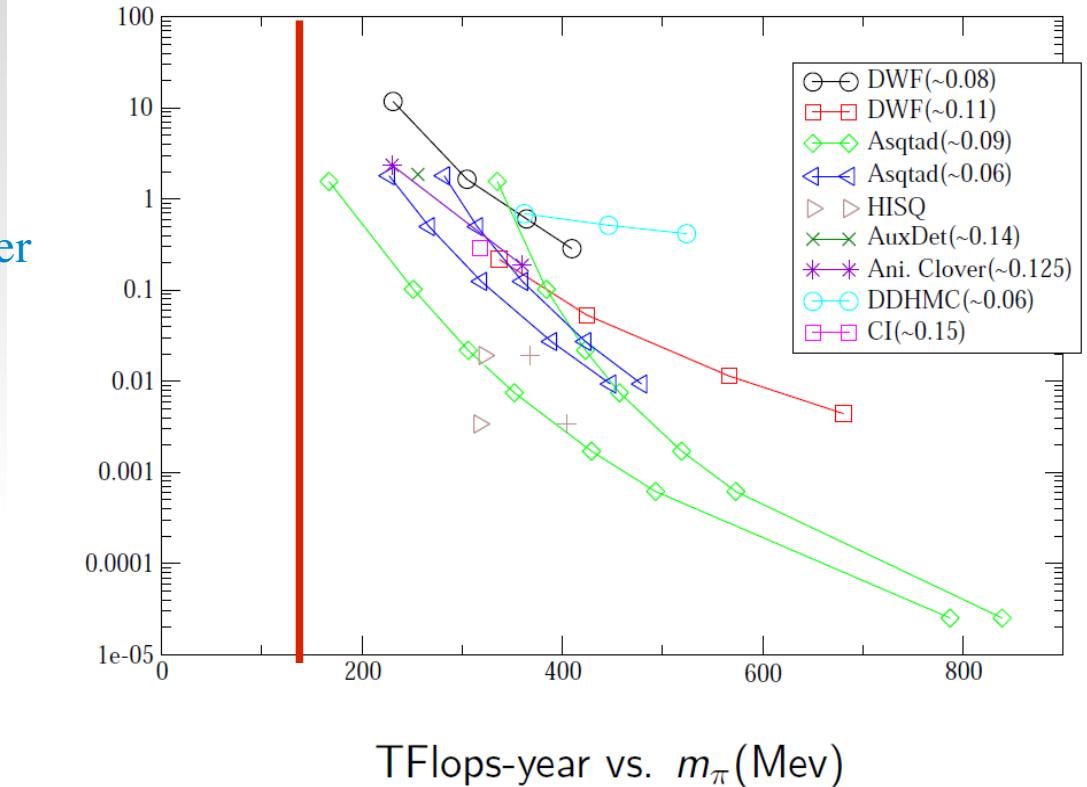
Dynamical QCD simulation

Algorithmic developments

- Rational Polynomial HMC for odd number of flavors
- multi-”time” step integrator separates UV and IR modes of quark fluctuations
 $\delta\tau_{\text{gluon}} \ll \delta\tau_{\text{quark,UV}} \ll \delta\tau_{\text{quark,IR}}$
- Preconditions by heavy quarks/domain decomposition for Dirac determinants

$$\det(D + m) = \det(D + \textcolor{red}{m}') \times \det(D + m)(D + \textcolor{red}{m}')^{-1}$$

[C. Jung Lattice09]



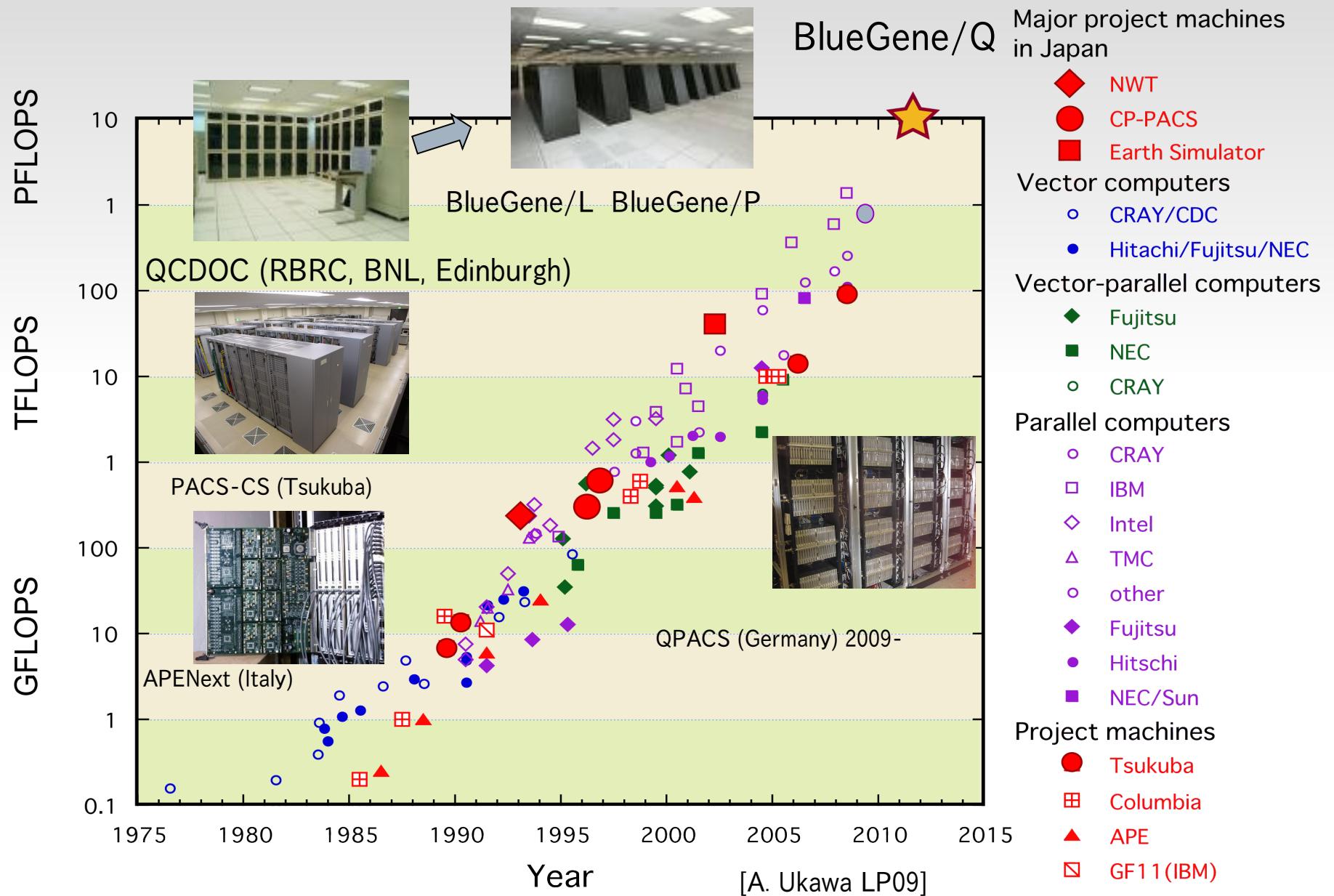
TFllops-year vs. m_π (Mev)

10^4 MD units, $m_\pi L = 4$, $T / L = 2$

Hardware developments

Needs chiral extrapolation from simulation points to experimental $m_\pi = 135$ MeV

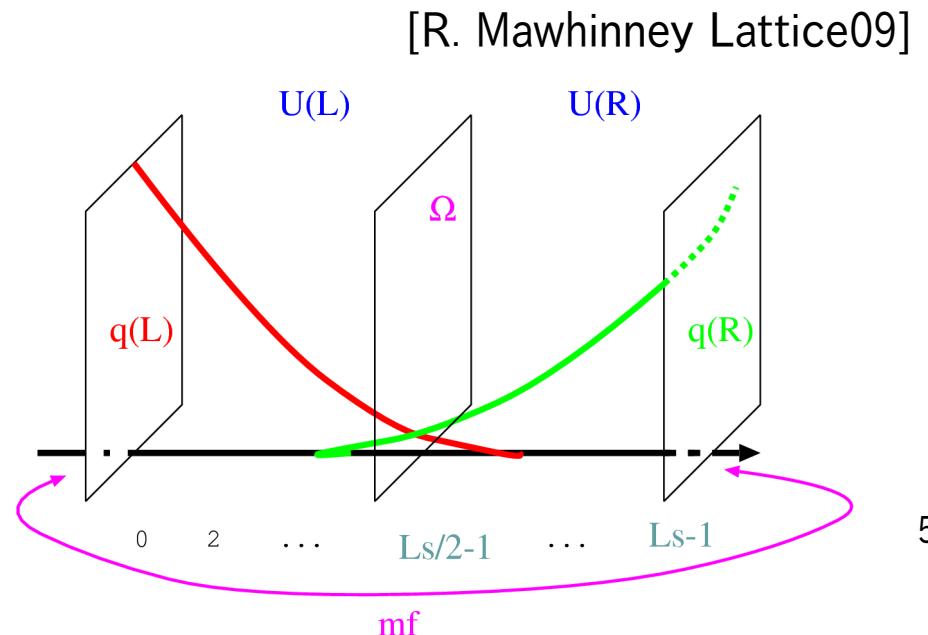
Computing Resources



Domain Wall Fermions

- Almost perfect chiral symmetric lattice quarks
 - Error from discretization is smaller $O(a^2) \sim O(1\%)$
 - Chiral extrapolation is simpler
- Vacuum polarization effects from up, down, strange ($N_f=2+1$) quarks are fully incorporated.
- Unitary theory

Volume	a^{-1} (GeV) m_{res}	$m_l m_1$	MD time units	Number of wall sources
$16^3 \times 32 \times 16$ $(1.82 \text{ fm})^3$	1.62(4) 0.00308(4)	(0.01, 0.04) (0.02, 0.04) (0.03, 0.04)	4015 4045 7600	
$24^3 \times 64 \times 16$ $(2.74 \text{ fm})^3$	1.75(3) 0.00315(2)	(0.005, 0.04) (0.01, 0.04) (0.02, 0.04) (0.03, 0.04)	8980 8540 2850 2813	2×203 2×178
$32^3 \times 64 \times 16$ $(\approx 3 \text{ fm})^3$	2.32(4) 0.000664	(0.004, 0.03) (0.006, 0.03) (0.008, 0.03)	6756 7220 5930	305 (P+A) 312 (P+A) 253 (P+A)
$32^3 \times 64 \times 32$ $(\approx 4.5 \text{ fm})^3$	≈ 1.4 ≈ 0.0018	(0.0042, 0.045)	3000	Being generated



RBC/UKQCD collaboration

BNL

C. Jung, T. Izubuchi, A. Soni,
R. Van de Water, O. Witzel

RIKEN-BNL Research Center (RBRC)

Y. Aoki, **K. Hashimoto**, T. Ishikawa,
S. Ohta

Columbia Univ.

N. Christ, **X-Y. Jin**, C. Kim, **M. Li**,
S. Li, **M. Lightman**, M. Lin, **Q. Liu**,
R. Mawhinney, **H. Peng**,

Univ. Connecticut

T. Blum, **S. Chowdhury**, R. Zhou

Univ. Virginia C. Dawson

FNAL E. Scholz

Harvard Univ. M. Clark

Univ. Edinburgh

D. Antonio, P. Boyle, **Paul Cooney**,
A. Hart, A. Kennedy, R. Kenway,
C. Kerry, C. Maynard, B. Pendleton,
J. Wennekers, J. Zanotti

Univ. Southampton

D. Brommel, M. Donnellan,
J. Flynn, A. Juttner, C. Sachrajda

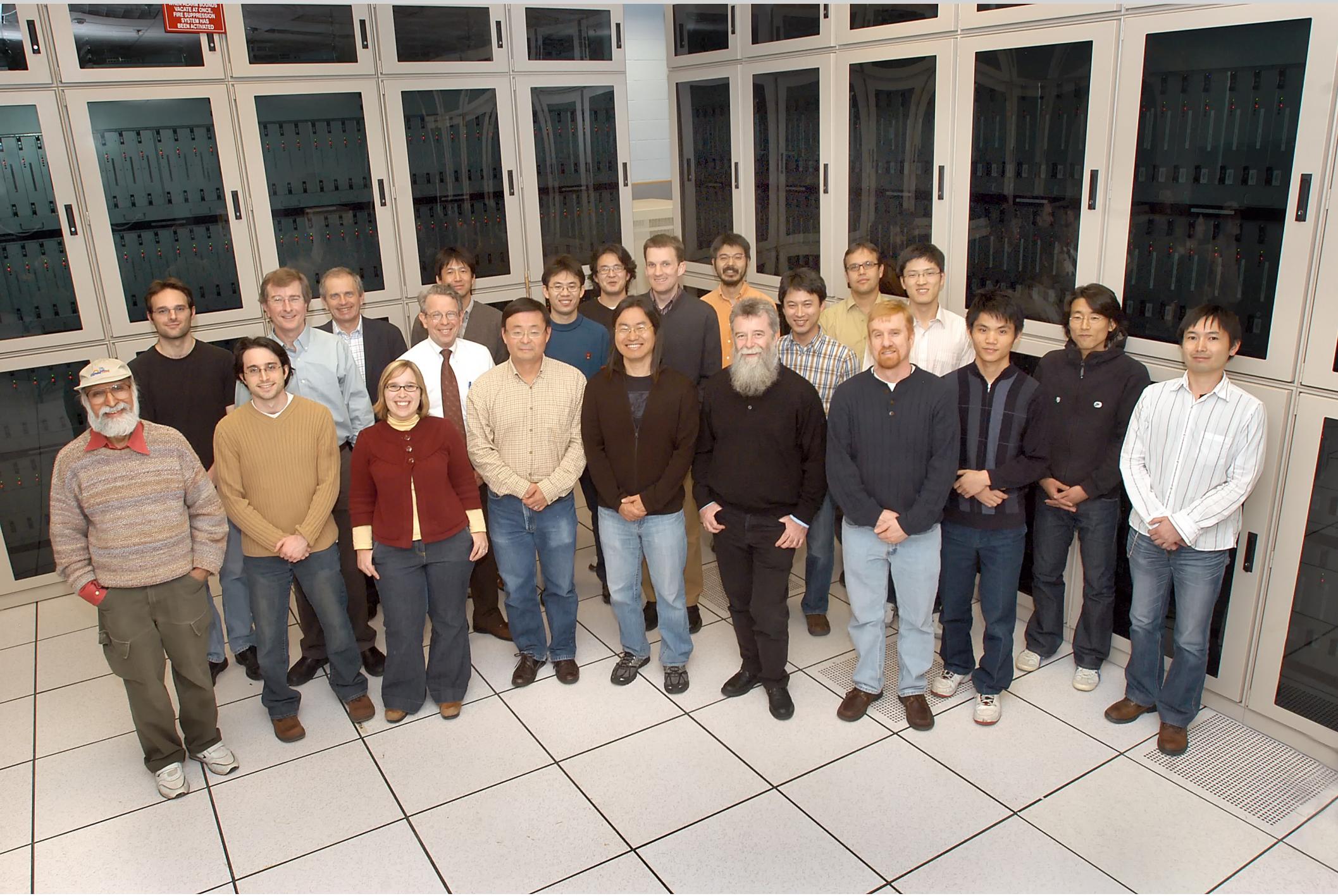
Univ. Swansea C. Allton

13 students, 16 PhD thesis

RIKEN(Japan) now has a program to support for
PhD students via TI. The use of this new avenue is
being explored with CU & U Conn.

2 plenary talks @ KAON09

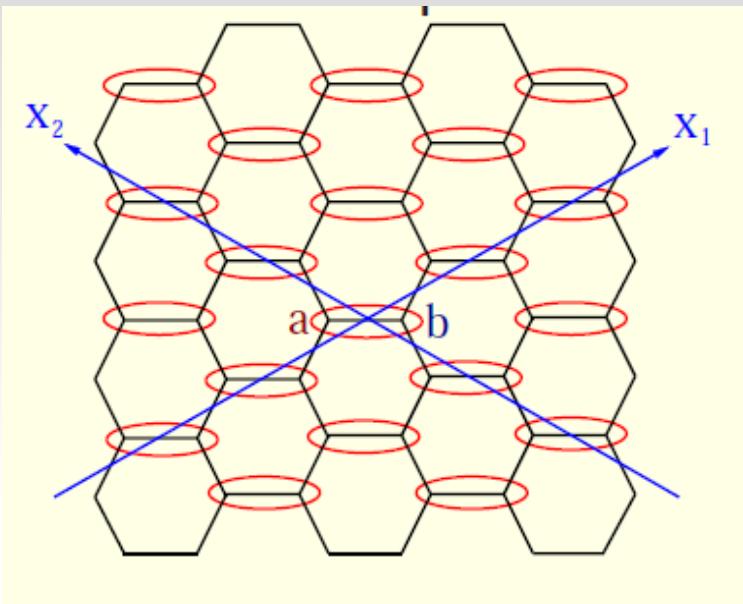
5 plenary talks @ LATTICE09



Menu of lattice QCD

- A new lattice Quark
- Hadron spectrum
- Chiral Perturbation Theory
SU(3), SU(2)
- Quark masses
- Renormalization/
Scheme matching
- Meson decay constants
- Electro-Magnetic(QED) Effects
- $g(\mu)-2$
- Proton decay
- Proton/Neutron Electric
Dipole moment
- Strange quark contents in
Nucleon (Dark matter detection)
- Flavor Physics, CKM,
Electro Weak Matrix Elements
 - f_K / f_π
 - $K \rightarrow \pi l \nu$ (Kl3)
 - K0-K0 mixing (B_K)
 - B, Charm physics
- Strong dynamics
(Technicolor), SUSY

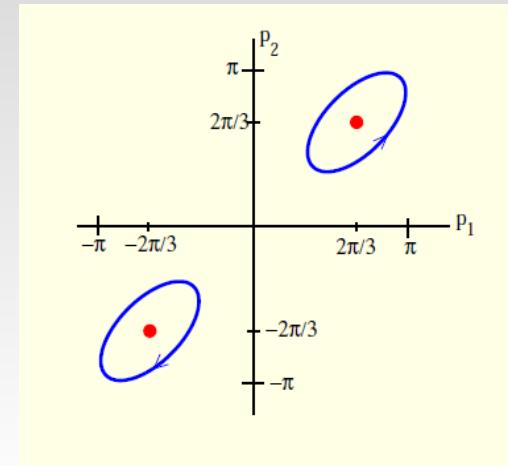
Graphene Fermion [M.Creutz]



H anti-commutes with

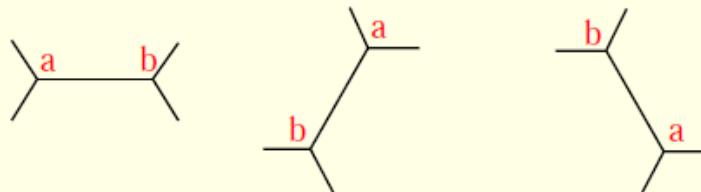
$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(Chiral symmetry)



- A two-flavor lattice quarks in 4 dim.
- One exact chiral symmetry
- Protects from additive mass renormalization (robust massless structure)
- Eigenvalues are purely imaginary in complex conjugate pairs
- Ultra local (fast to simulate)

$$z = 1 + e^{-ip_1} + e^{+ip_2}$$



$$\tilde{H}(p_1, p_2) = K \begin{pmatrix} 0 & z \\ z^* & 0 \end{pmatrix}$$

QCD+QED simulation

[R. Zhou, S. Uno, T. Blum, T. Doi, M. Hayakawa, Tl, N. Yamada,]

- Up down quark has different electric charge and masses

→ Breaking of **isospin symmetry**

- Isospin breaking effects are accurately measured experimentally

$$\Delta m_\pi = m_{\pi^\pm} - m_{\pi^0} = 4.5936(5) \text{ MeV}, m_N - m_P = 1.2933317(5) \text{ MeV}$$

- Quark masses.** Is up quark massless possible ?

Can it explain the very small Neutron Electric Dipole Moment ? (**Strong CP problem**)

- Compute Pion/Kaon using Nf=2+1 DWF QCD + QED

Requiring $m_q < 40 \text{ MeV}$ (70MeV), 48 (120) partially quenched data points for PS meson survive

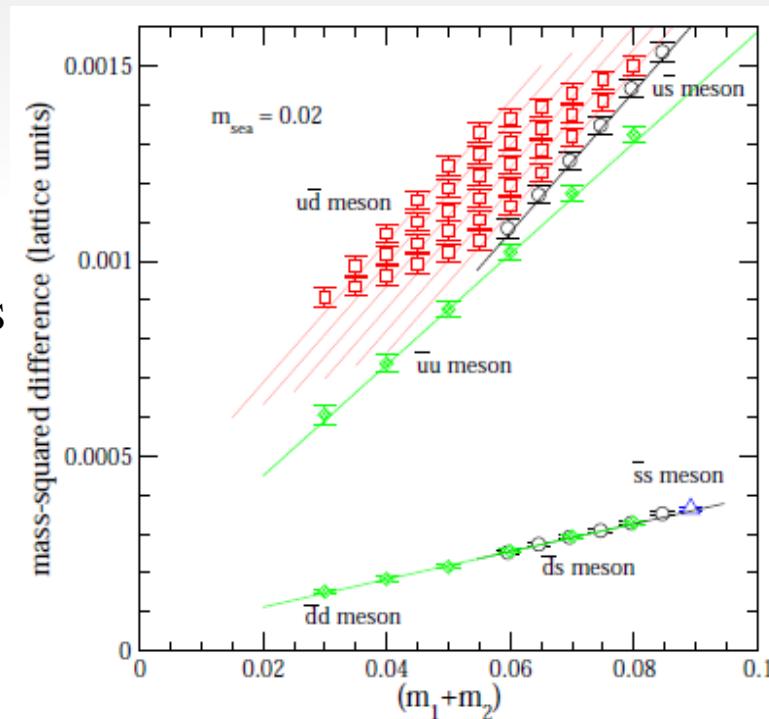
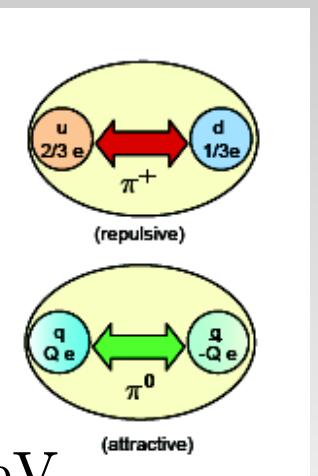
- Fit to chiral perturbation theory with EM (**SU(3)+EM** and **SU(2)+Kaon+EM**) to extract quark masses.

- Chiral symmetry is essential to define unambiguous quark massless points.

- Input: $M_{PS}(m_{up}, 2/3, m_{dwn}, -1/3) = 139.57018(35) \text{ MeV}$

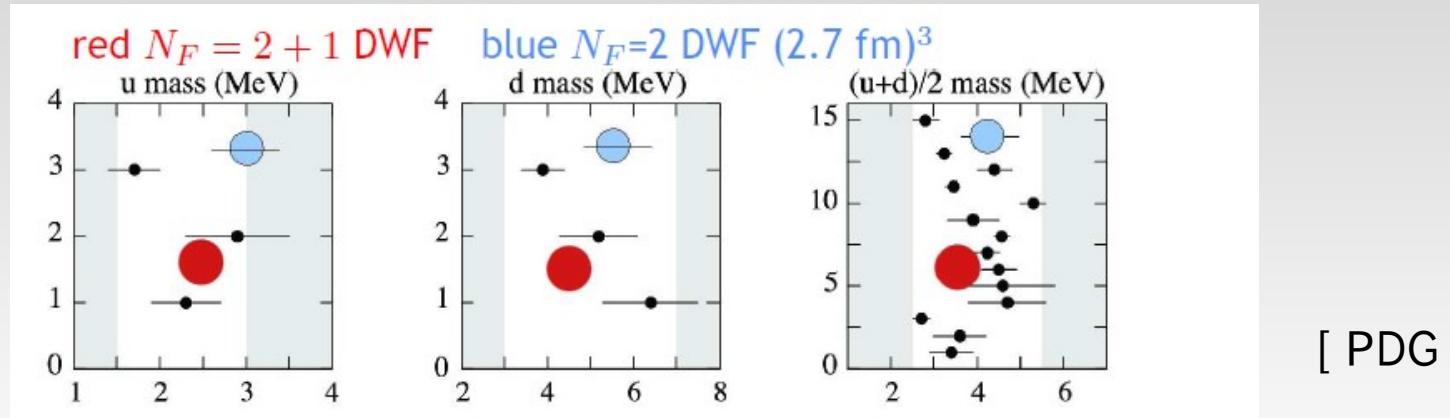
$$M_{PS}(m_{up}, 2/3, m_{str}, -1/3) = 493.673(14) \text{ MeV}$$

$$M_{PS}(m_{dwn}, -1/3, m_{str}, -1/3) = 497.614(24) \text{ MeV}$$



Quark mass results

- QCD+QED simulation results (statistical error only, systematic error: in progress)

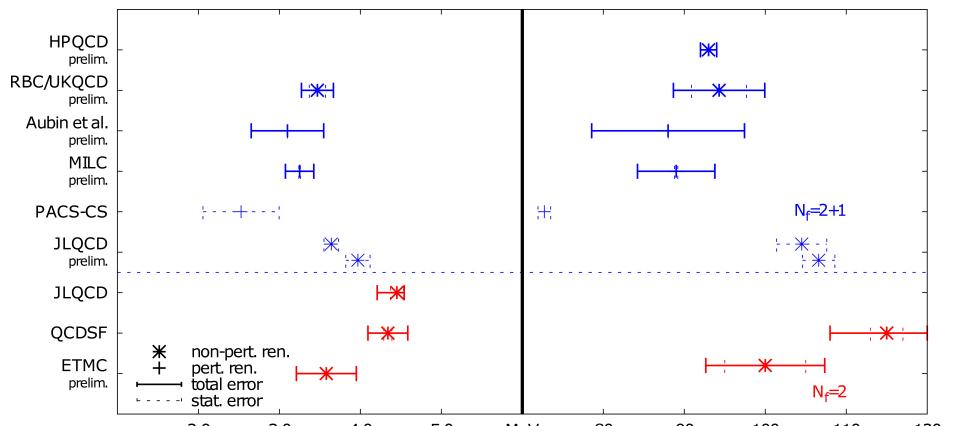


- Origin of charge splitting of Kaons

$$M_{K^\pm} - M_{K^0} = -3.937(29) \text{ MeV},$$

$$\begin{aligned} \Delta M(m_{up} - m_{dwn}) &= -5.7(1) \text{ MeV} \\ \Delta M(q_u - q_d) &= 1.8(1) \text{ MeV} \end{aligned}$$

- Status of quark mass determinations from lattice QCD
(In this plot, QED effects are estimated by Dashen's theorem)



Nf=2+1 results

$$\bar{m}(\overline{\text{MS}}, 2\text{GeV}) = (m_u + m_d)/2$$

$$m_s(\overline{\text{MS}}, 2\text{GeV})$$

Nf=2 results

[E. Scholtz Lattice09]

New Renormalization Scheme

[C. Sturm, Y. Aoki, N. Christ, TI, C. Sachrajda, A. Soni]

- Match the normalization of operator on lattice and in continuum theory (MS) in RI/SMOM scheme

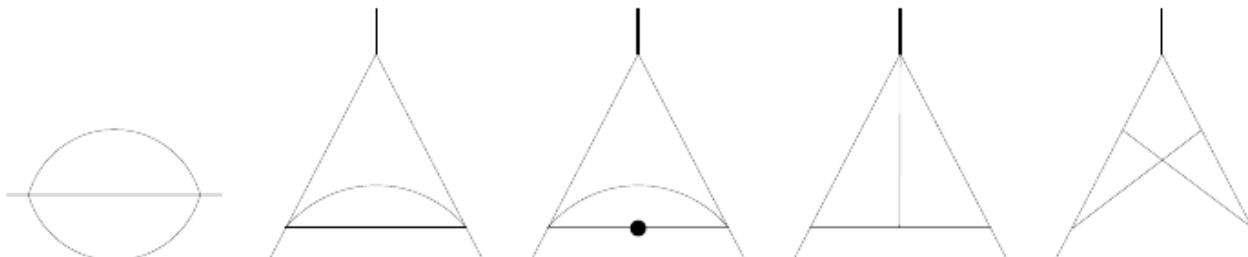
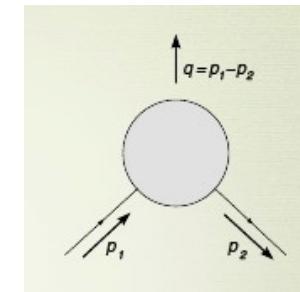
calculat the 3 pt amplitudes for {a momentum configuration on lattice non-perturbatively, same mom. Configuration in continuum, and in MS}



- We find symmetric momentum (SMOM) configuration is useful to reduce one of the dominant systematic error due to IR effects, $\sim 10\% \rightarrow \sim 5\%$ (PT dominant), of central values for quark masses [Y. Aoki Lattice09]

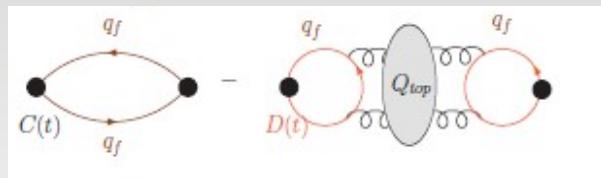
$$(q^2 = p_1^2 = p_2^2)$$

- Four quark operator for B_K
- Now the two loop matching calculation is in progress

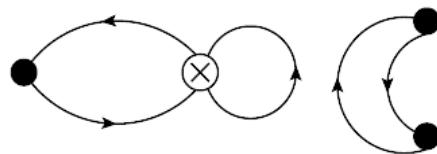


[L. Almeida, C. Sturm, TI]

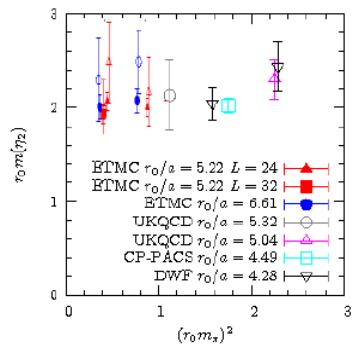
- Flavor singlet meson η' and disconnected diagram (U(1) A problem)



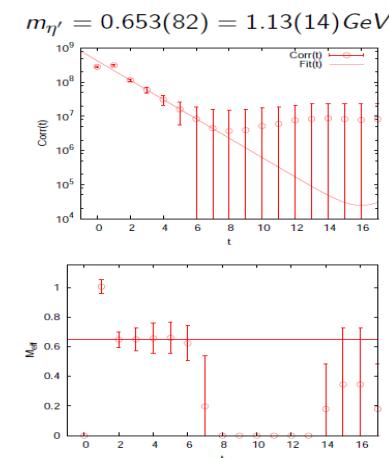
- Challenging yet important calculation (K $\rightarrow\pi\pi$, g(μ)-2)



- $\sim 15\%$ stat. error on Nf=2 and 2+1 DWF ensemble

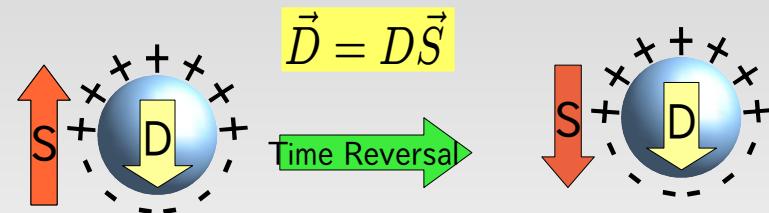


[Nf=2 K. Hashimoto thesis]



[Nf=2+1 Q. Liu Lattice2009]

- Electric Dipole Moment (EDM) of Proton/Neutron (Strong CP problem)



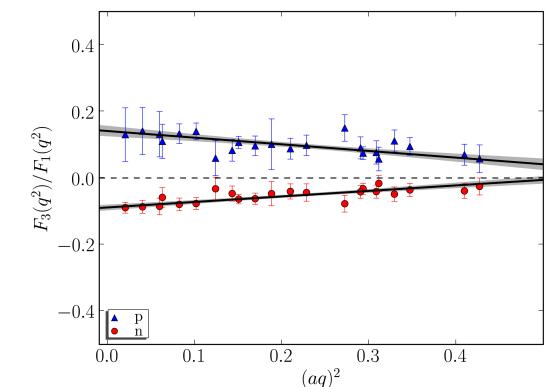
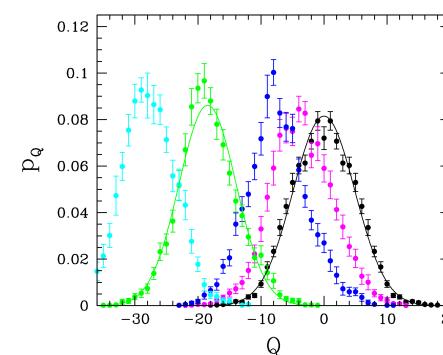
- vacuum angle θ , is implemented on lattice with analytical continuation:

$$\theta \rightarrow -i\theta$$

- Topological charge distribution & EDM

$$D_n = \lim_{q^2 \rightarrow 0} \frac{e}{2m_N} F_3(q^2)$$

$$\theta = 1.5, 1.0, 0.4, 0.2, 0.0$$



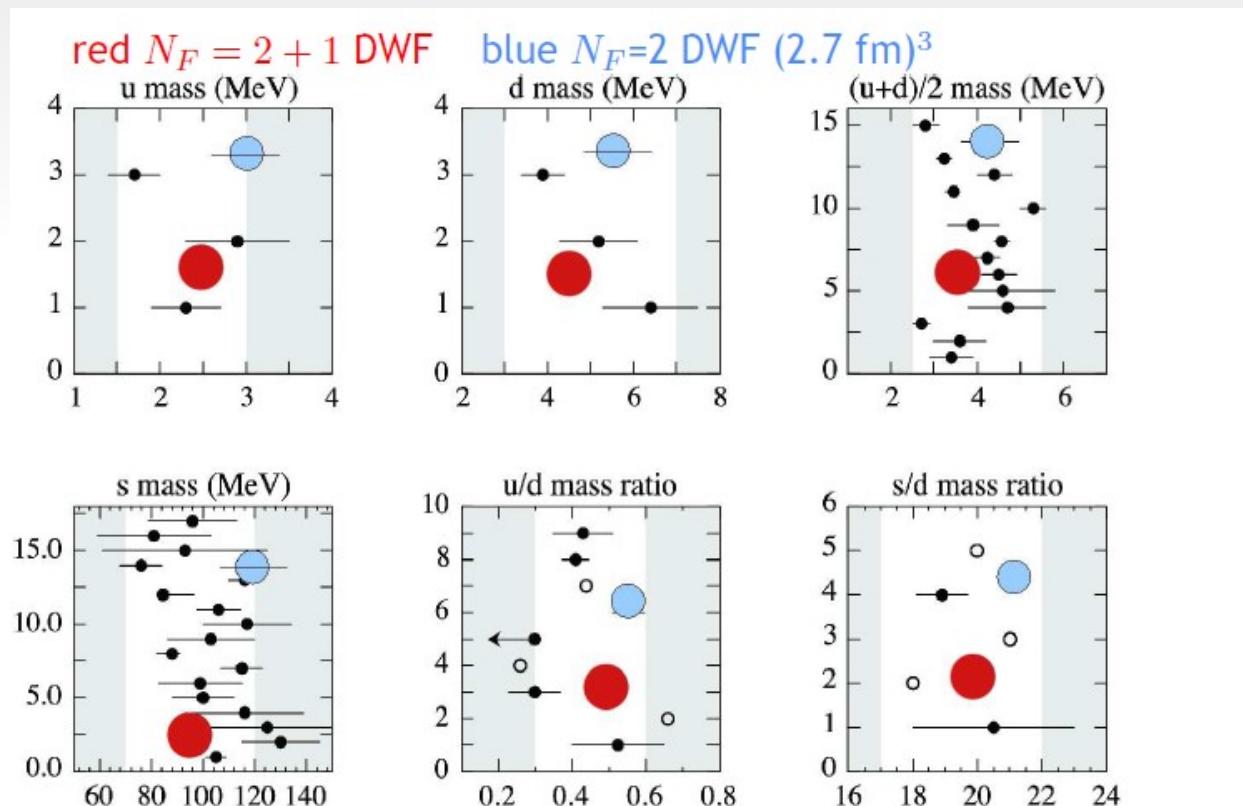
Summary

- Lattice QCD is becoming a practical tool for non-perturbative calculation for particle physics from the first principle
(bridge b/w experiments and theories)
- DWF, preserves chiral symmetry, is optimal for (Weak) Matrix elements (\rightarrow R. Van de Water's talk), which are necessary ingredients for precise check for the standard model of the particle physics and beyond.
- Realistic calculation for DWF $N_f=2+1$ is being carried out thanks to many developments of theories, hardwares, and algorithms.
- Sample of Lattice activities:
 - A new chiral quark formalism
 - Hadron spectrum and quark masses from QCD+QED simulation.
 - New renormalization schemes RI/SMOM
 - η' and disconnected diagrams
 - Proton Neutron EDM using imaginary θ

Appendix

QCD+QED results

lat	m_q range	m_u	m_d	m_s	m_u/m_d	m_s/m_{ud}
$(2.7 \text{ fm})^3 \text{ SU}(3)_\infty$	$\leq 40 \text{ MeV}$	2.48(18)	4.77(30)	95(7)	0.52(3)	26.3(6)
$(2.7 \text{ fm})^3$	$\leq 70 \text{ MeV}$	2.50(18)	4.81(30)	95(8)	0.52(3)	26.1(6)
$(1.8 \text{ fm})^3$	$\leq 70 \text{ MeV}$	2.64(19)	4.81(32)	95(9)	0.55(4)	25.5(8)
$(2.7 \text{ fm})^3 \text{ SU}(2)_\infty$	$\leq 70 \text{ MeV}$	2.24(16)	4.62(24)	101(5)		



(Only statistical errors are shown).

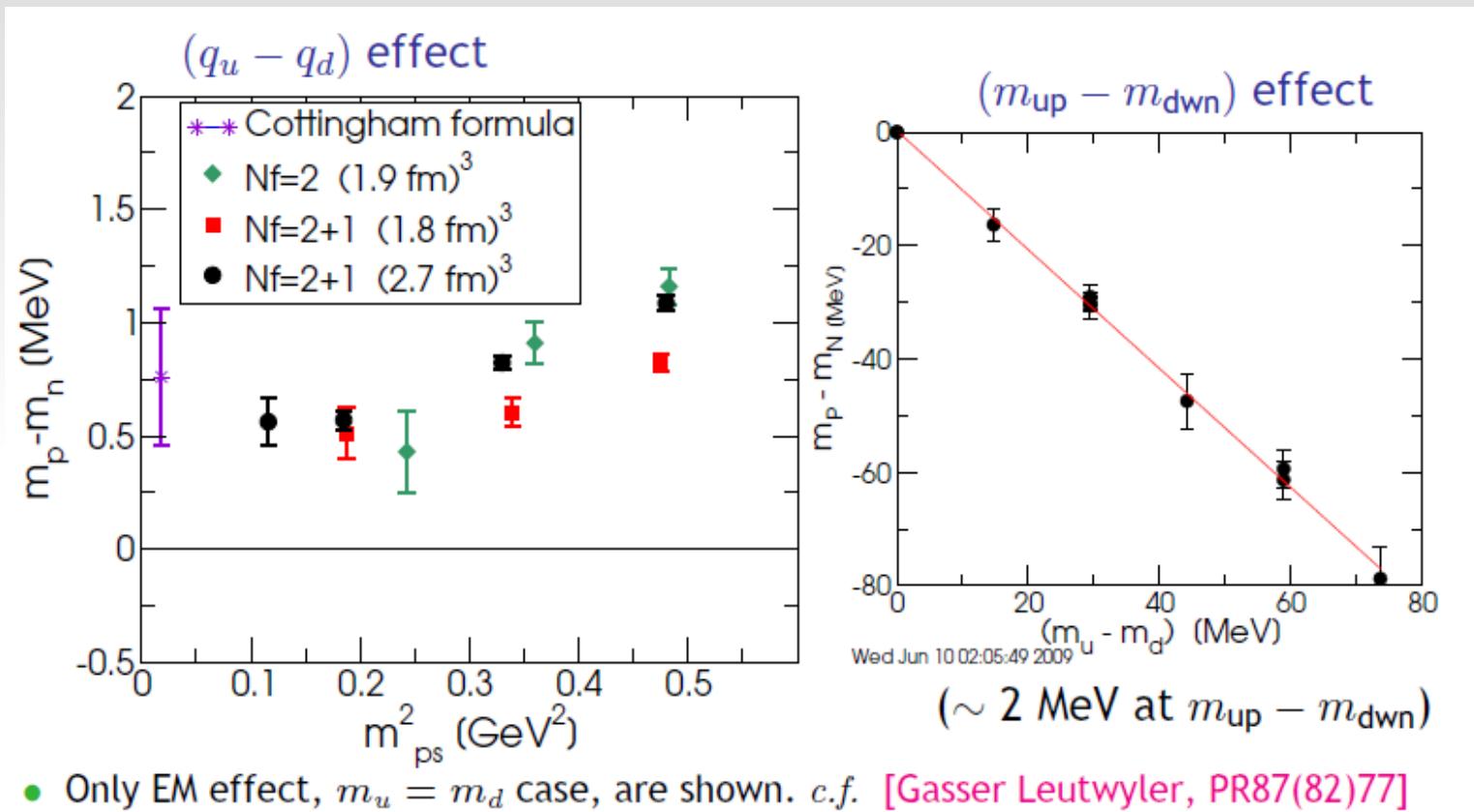
$$M_{K^\pm} - M_{K^0} = -3.937(29) \text{ MeV},$$

$$\Delta M(m_{up} - m_{down}) = -5.7(1) \text{ MeV}$$

$$\Delta M(q_u - q_d) = 1.8(1) \text{ MeV}$$

Systematic error is being estimated

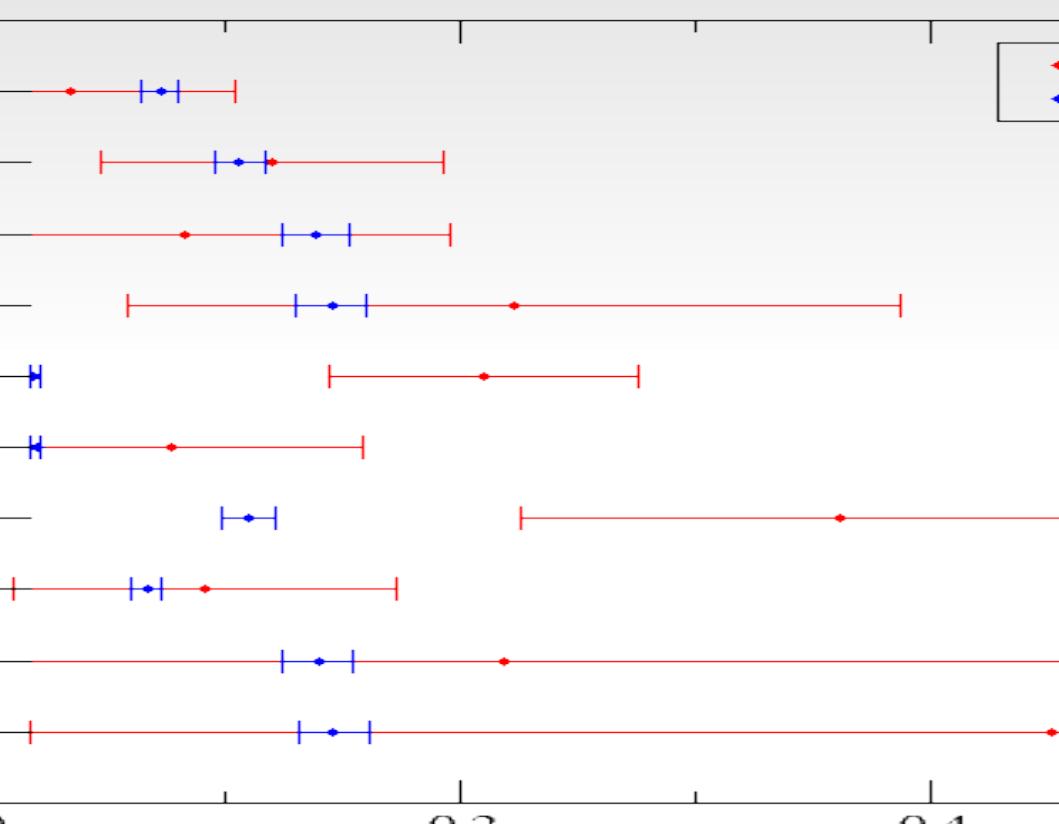
Proton/Neutron splitting



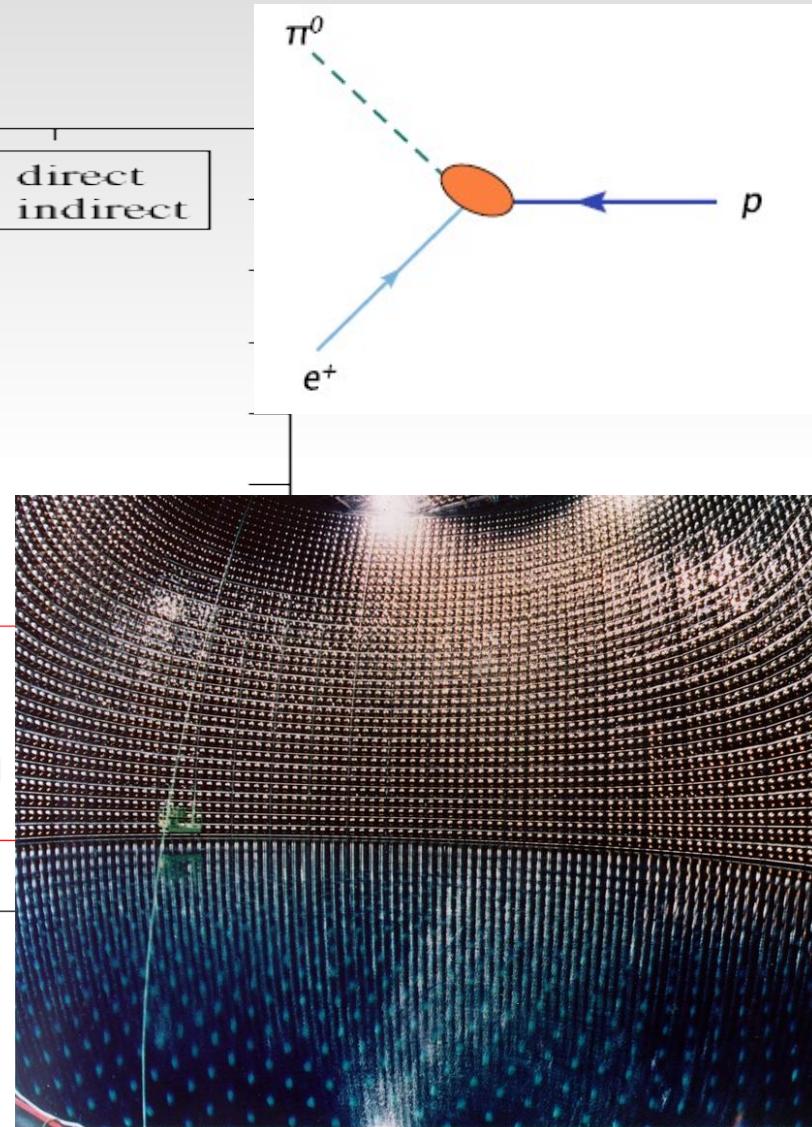
$$M_N - M_p|_{\text{EM}} = -0.76(30) \text{ MeV}$$

$$M_N - M_p|_{\text{quark mass}} = 2.05(30) \text{ MeV}$$

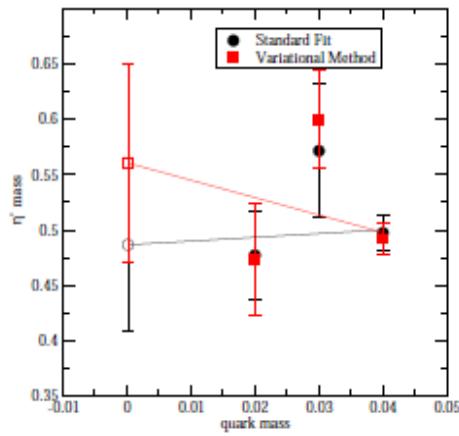
Proton decay matrix elements [Y. Aoki, C. Maynard, et. al.]



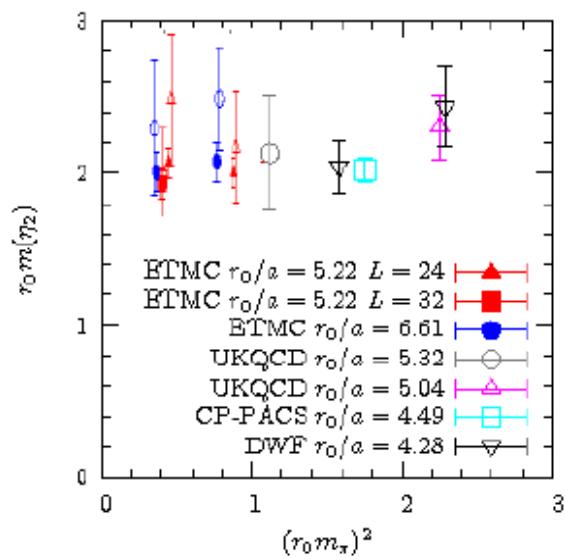
direct
indirect



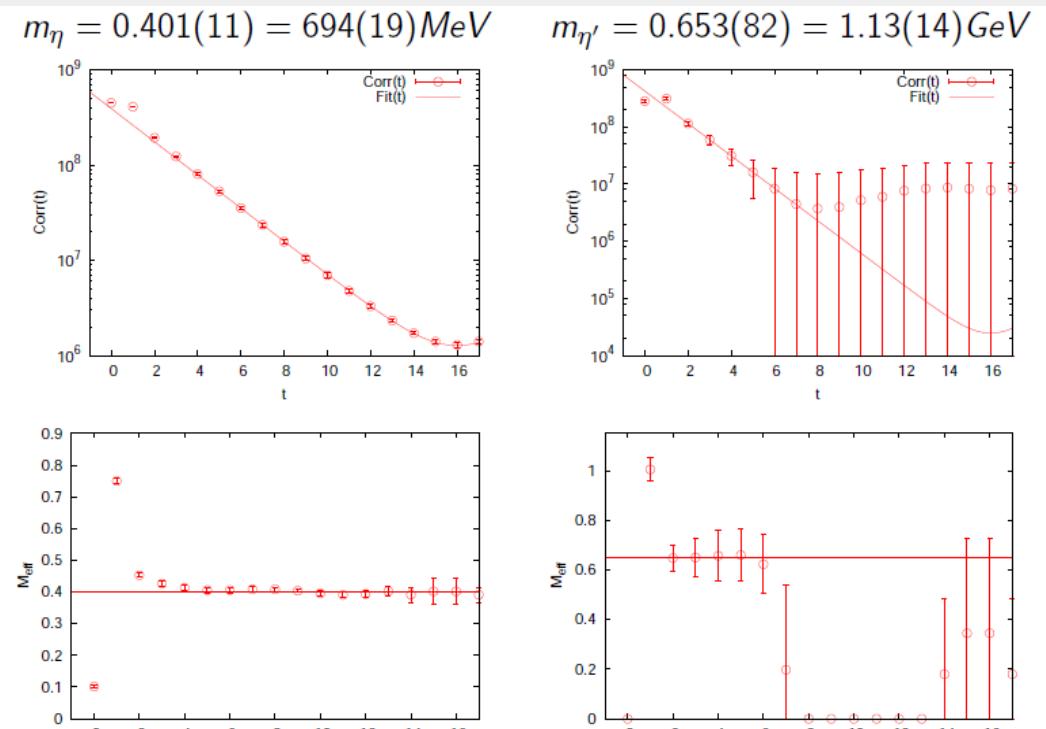
Eta' results for Nf=2 & 2+1



$m_{\eta'}$	$m_{\eta'}^{\text{phys}} [\text{MeV}]$	$m_{\eta'} r_0$	Fit and chiral extrapolation
0.480(78)	738(121)	2.05(33)	(Standard) AWTI
0.487(78)	748(120)	2.08(33)	(Standard) linear
0.532(82)	819(127)	2.28(35)	(Variational) AWTI
0.560(89)	862(130)	2.40(36)	(Variational) linear



[Nf=2 K. Hashimoto]

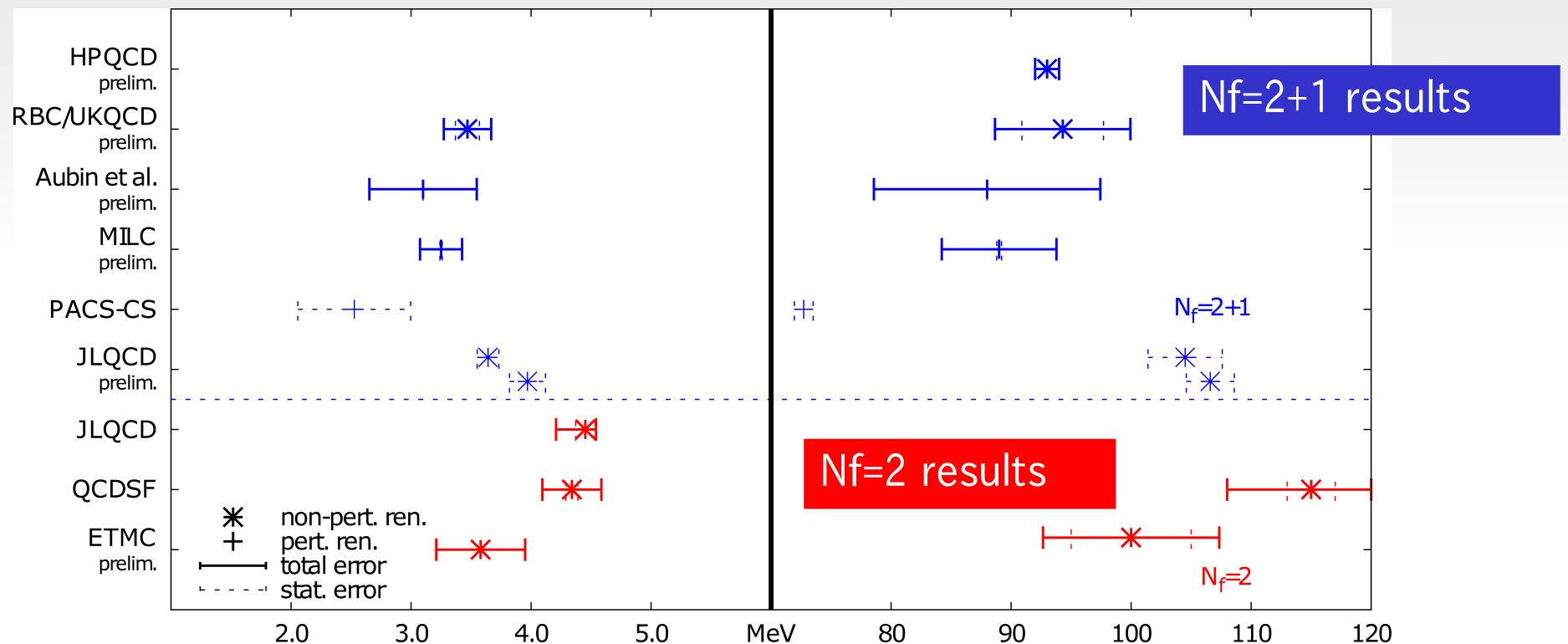


[Nf=2+1 Q. Liu Lattice2009]

Status of Light quark masses

(w/o QED calculation)

QED effects are removed using Dashen's theorem



$$\bar{m}(\overline{\text{MS}}, 2\text{GeV}) = (m_u + m_d)/2$$

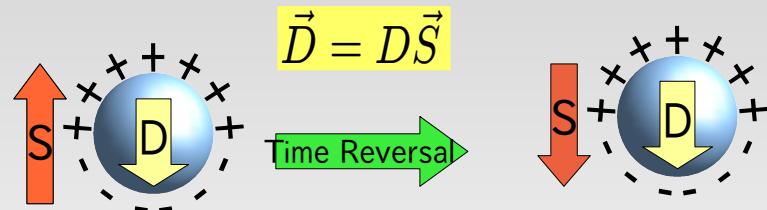
$$m_s(\overline{\text{MS}}, 2\text{GeV})$$

[E. Scholtz Lattice09]

Proton/Neutron EDM on Lattice

Permanent Electric Dipole Moment (EDM) is a signature of **CP** (or Time reversal) symmetry violation.

A source of **CP** violation :



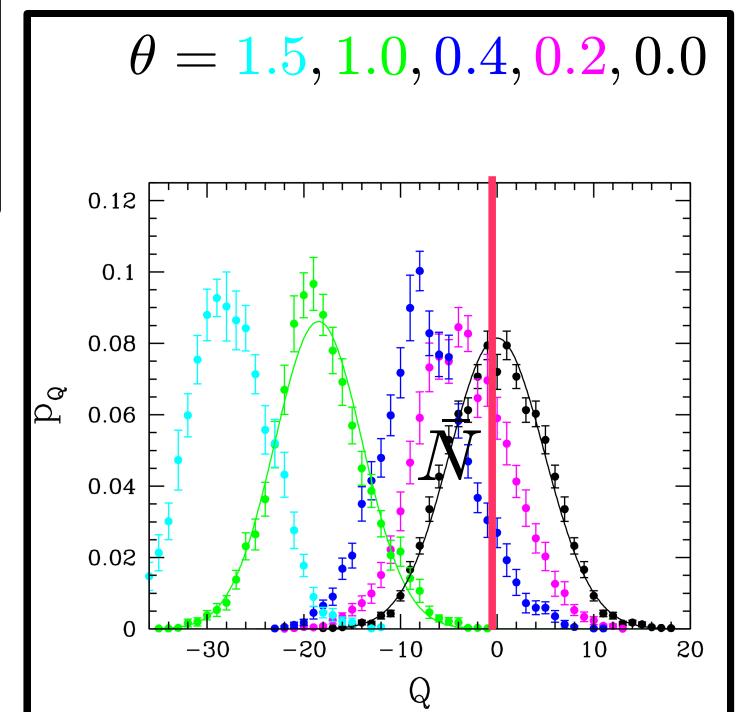
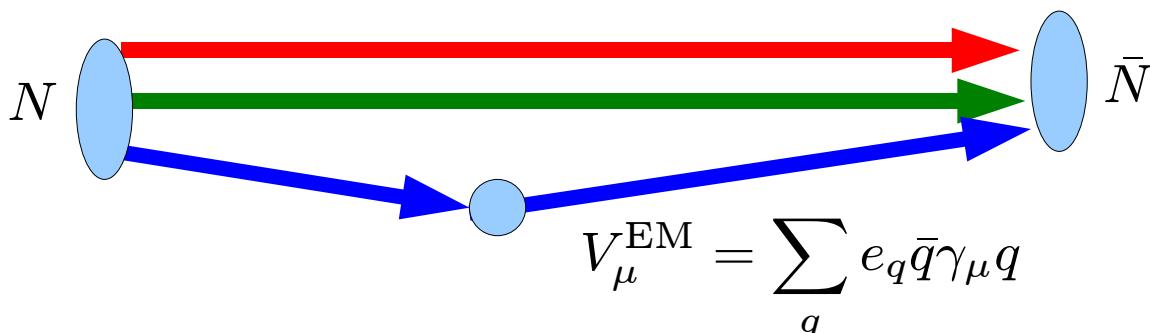
Strong CP: vacuum angle θ , is implemented on lattice with analytically continued to pure imaginary (Monte Carlo simulation)

$$\theta \rightarrow -i\theta$$

EDM is measured through the electric form factor $F_3(q^2)$

$$D_n = \lim_{q^2 \rightarrow 0} \frac{e}{\gamma m_N} F_\gamma(q^\gamma)$$

$$\begin{aligned} \left\langle N_s(\mathbf{p}') | V_\mu^{EM}(\mathbf{q}) | \bar{N}_s(\mathbf{p}) \right\rangle_\theta &= F_1(q^2) \gamma_\mu + F_2(q^2) \frac{q_\nu \sigma_{\mu\nu}}{2m_N} \\ &+ i\theta F_3(q^2) \frac{q_\nu \sigma_{\mu\nu} \gamma_5}{2m_N} + \dots, \quad q = p' - p \end{aligned}$$



Results of F_3

valence theta = sea theta = 0.2 (left) and 0.4 (right)

Dipole ansatz

$$F_3^\theta(q^2) = \frac{F_3^\theta(0)}{(1 + q^2/M^2)^2}$$

DWF simulation is planned for remove systematic errors (valence theta)

